2019 AMERICAN MATHEMATICS COMPETITION 10
2019 年美国十年级数学竞赛（AMC10）

1.What is the value of

$$2^{\left(0^{\left(1^{9}\right)}\right)}+\left(\left(2^{0}\right)^{1}\right)^{9}?$$

上述表达式的结果是多少？

$$2^{\left(0^{\left(1^{9}\right)}\right)}+\left(\left(2^{0}\right)^{1}\right)^{9}?$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. What is the hundreds digit of $(20! - 15!)$?

$(20! - 15!)$的百位数字是多少？

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

3. Ana and Bonita were born on the same date in different years, n years apart. Last year Ana was 5 times as old as Bonita. This year Ana's age is the square of Bonita's age. What is n ?

Ana 和 Bonita 出生在不同年份的同一天，相隔 n 年。去年 Ana 的年龄是 Bonita 年龄的 5 倍。今年 Ana 的年龄是 Bonita 年龄的平方。问 n 是多少？

(A) 3 **(B)** 5 **(C)** 9 **(D)** 12 **(E)** 15

4. A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

一个盒子中有 28 个红球，20 个绿球，19 个黄球，13 个蓝球，11 个白球，和 9 个黑球。为了

保证至少取出 15 个单一颜色的球，在不允许放回重取的情况下，必须从盒子里取出的球的数量最少是多少个？

(A) 75 **(B)** 76 **(C)** 79 **(D)** 84 **(E)** 91

5. What is the greatest number of consecutive integers whose sum is 45 ?

最多可以有多少个连续整数，它们的总和是 45?

(A) 9 **(B)** 25 **(C)** 45 **(D)** 90 **(E)** 120

6. For how many of the following types of quadrilaterals does there exist a point in the plane of the quadrilateral that is equidistant from all four vertices of the quadrilateral?

- a square
- a rectangle that is not a square
- a rhombus that is not a square
- a parallelogram that is not a rectangle or a rhombus
- an isosceles trapezoid that is not a parallelogram

在以下类型的四边形中有多少种，在四边形所在平面上存在一个与四边形的所有四个顶点等距的点？

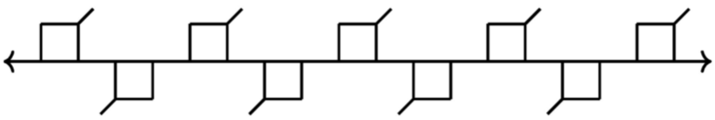
- 正方形
- 不是正方形的长方形
- 不是正方形的菱形
- 不是长方形或菱形的平行四边形
- 不是平行四边形的等腰梯形

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

7. Two lines with slopes $\frac{1}{2}$ and 2 intersect at $(2,2)$. What is the area of the triangle enclosed by these two lines and the line $x+y=10$?
两条斜率分别为 1/2 和 2 的直线相交于(2, 2)。问由这两条直线和直线 x + y = 10 所框出的三角形的面积是多少？

(A) 4 **(B)** $4\sqrt{2}$ **(C)** 6 **(D)** 8 **(E)** $6\sqrt{2}$

8. The figure below shows line *A* with a regular, infinite, recurring pattern of squares and line segments.
下图显示了由直线 *A*，以及按照规则、无限的、重复出现的正方形和线段组成的图形。



How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?
在下面的四种关于该图形所在平面的刚性变换中，有几种是除了恒等变换外，能够把上述图形变到自身的？

- some rotation around a point on line *A* 围绕直线 *A* 上的某个点的旋转
- some translation in the direction parallel to line *A* 沿着平行于直线 *A* 的方向的某个平

移

- the reflection across line *A* 关于直线 *A* 的反射

· some reflection across a line perpendicular to line A 关于某条垂直于直线 A 的直线的反射

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. What is the greatest three-digit positive integer n for which the sum of the first n positive integers is *not* a divisor of the product of the first n positive integers?

前 n 个正整数的总和不是前 n 个正整数的乘积的约数的最大三位正整数 n 是多少？

(A) 995 **(B)** 996 **(C)** 997 **(D)** 998 **(E)** 999

10. A rectangular floor that is 10 feet wide and 17 feet long is tiled with 170 one-foot square tiles. A bug walks from one corner to the opposite corner in a straight line. Including the first and last tile, how many tiles does the bug visit?

一块长方形地板的宽是 10 英尺，长是 17 英尺，铺有 170 块一英尺见方的瓷砖。一只虫子从一个角落沿直线走到相对的另一个角落。包括第一块和最后一块瓷砖，虫子一共经过了多少块瓷砖？

(A) 17 **(B)** 25 **(C)** 26 **(D)** 27 **(E)** 28

11. How many positive integer divisors of 201^9 are perfect squares or perfect cubes (or both)?

在 201^9 的正整数约数中， 是完全平方数或者是完全立方数（或者兼具这两个特点）的数有多少个？

(A) 32 **(B)** 36 **(C)** 37 **(D)** 39 **(E)** 41

12. Melanie computes the mean μ, the median M, and modes of the 365 values that are the dates in the months of 2019. Thus her data consist of 12 1s, 12 2s, · · · , 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

Melanie 计算 2019 年的各个月的日期这 365 个数值的平均数 μ，中位数 M 以及众数。因此，她的数据包括 12 个 1，12 个 2，…，12 个 28，11 个 29，11 个 30 ，以及 7 个 31。设 d 为众数的中位数。下列哪个论断是正确的？

(A) $\mu < d < M$ **(B)** $M < d < \mu$ **(C)** $d = M = \mu$ **(D)** $d < M < \mu$ **(E)** $d < \mu < M$

13. Let $\triangle ABC$ be an isosceles triangle with $BC = AC$ and $\angle ACB = 40°$. Construct the circle with diameter BC, and let D and E be the other intersection points of the circle with the sides AC and AB, respectively. Let F be the intersection of the diagonals of the quadrilateral $BCDE$. What is the degree measure of $\angle BFC$?

设 OABC 为等腰三角形，$BC = AC$ 并且$\angle ACB = 40°$。构造直径为 BC 的圆，D 和 E 分别为圆与边 AC 和 AB 的另一个交点。设 F 为四边形 $BCDE$ 的对角线的交点。 $\angle BFC$ 是多少度？

(A) 90 **(B)** 100 **(C)** 105 **(D)** 110 **(E)** 120

14. For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N?

考虑平面上四条不同的直线构成的集合，在其中两条或更多条直线上的不同的点恰好有 N 个。问 N 的所有可能值的总和是多少？

(A) 14 **(B)** 16 **(C)** 18 **(D)** 19 **(E)** 21

15. A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = 3/7$. and

$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

for all $n \geq 3$. Then a_{2019} can be written as p/q， where p and q are relatively prime positive integers. What is $p + q$?

递归的定义数列：$a_1 = 1$，$a_2 = \underline{3}$ ，并且对于所有 $n \geq 3$，

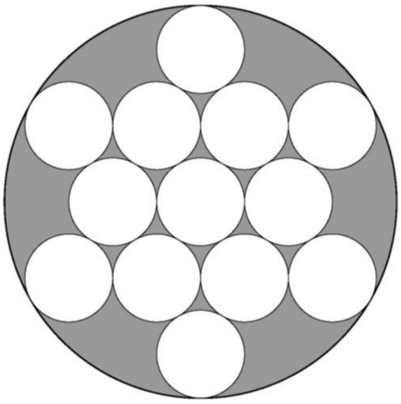
$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

a_{2019} 可以写成 p/q ，其中 p 和 q 是互质的正整数。问 $p + q$ 是多少？

(A) 2020 **(B)** 4039 **(C)** 6057 **(D)** 6061 **(E)** 8078

16. The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all of the circles of radius 1?

下图显示了 13 个半径为 1 的圆在一个较大圆内。所有相交的点都是切点。在较大的圆内，但在所有半径为 1 的圆外的阴影部分的面积是多少？



(A) $4\pi\sqrt{3}$ **(B)** 7π **(C)** $\pi\left(3\sqrt{3}+2\right)$ **(D)** $10\pi\left(\sqrt{3}-1\right)$ **(E)** $\pi\left(\sqrt{3}+6\right)$

17. A child builds towers using identically shaped cubes of different colors. How many different towers with a height of 8 cubes can the child build with 2 red cubes, 3 blue cubes, and 4 green cubes? (One cube will be left out.)

小朋友用大小一样、颜色不同的立方体来搭建塔。小朋友用 2 个红色立方体，3 个蓝色立方体和 4 个绿色立方体可以构建出多少个不同的有 8 个立方体高的塔？ （有一个立方体没有用到。）

(A) 24 **(B)** 288 **(C)** 312 **(D)** 1, 260 **(E)** 40, 320

18. For some positive integer k, the repeating base-k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323..._k$. What is k?
对于某个正整数 k，十进制表示中的分数 7/15，在 k 进制下的循环小数表示为 $0.\overline{23}_k = 0.232323..._k$，问 k 是多少？

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

19. What is the least possible value of
$(x + 1)(x + 2)(x + 3)(x + 4)+ 2019$,
where x is a real number?
x 是实数，下式的最小可能值是多少？
$(x + 1)(x + 2)(x + 3)(x + 4)+ 2019$
(A) 2017 **(B)** 2018 **(C)** 2019 **(D)** 2020 **(E)** 2021

20. The numbers 1, 2, ..., 9 are randomly placed into the 9 squares of a 3 × 3 grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?
数字 1，2，...，9 被随机放入 3× 3 方格表的 9 个单位正方形中。每个单位正方形里有一个数字，并且每个数字均使用一次。那么每个横行与每个竖列中的数字之和均是奇数的概率是多少？
(A) $\frac{1}{21}$ **(B)** $\frac{1}{14}$ **(C)** $\frac{5}{63}$ **(D)** $\frac{2}{21}$ **(E)** $\frac{1}{7}$

21. A sphere with center O has radius 6. A triangle with sides of length 15, 15, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?
中心为 O 的球体的半径是 6。在空间中有一个边长分别为 15、15、24 的三角形，它的每条边都与球体相切。问 O 和该三角形确定的平面之间的距离是多少？

(A) $2\sqrt{3}$ **(B)** 4 **(C)** $3\sqrt{2}$ **(D)** $2\sqrt{5}$ **(E)** 5

22. Real numbers between 0 and 1, inclusive, are chosen in the following manner. A fair coin is flipped. If it lands heads, then it is flipped again and the chosen number is 0 if the second flip is heads, and 1 if the second flip is tails. On the other hand, if the first coin flip is tails, then the number is chosen uniformly at random from the closed interval

[0, 1]. Two random numbers x and y are chosen independently in this manner. What is the probability that $|x-y|>1/2$?

0 与 1 之间（包括两端）的实数按下列方式选择。抛掷一枚均匀的硬币。如果是正面向上，那么它会被再次抛掷，如果第二次是正面向上，则选择数 0，如果第二次是背面向上，则选择数 1。另一方面，如果第一次是背面向上，则从闭区间[0, 1]中按随机均匀分布选择一个数以这种方式独立选择两个数 x 和 y。那么|x − y| >1/2 的概率是多少？

(A) $\frac{1}{3}$ **(B)** $\frac{7}{16}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{2}{3}$

23. Travis has to babysit the terrible Thompson triplets. Knowing that they love big numbers, Travis devises a counting game for them. First Tadd will say the number 1, then Todd must say the next two numbers (2 and 3), then Tucker must say the next three numbers (4, 5, 6), then Tadd must say the next four numbers (7, 8, 9, 10), and the process continues to rotate through the three children in order, each saying one more number than the pre- vious child did, until the number 10, 000 is reached.What is the 2019th number said by Tadd?

Travis 需要照看 Thompson 不好管的三胞胎。Travis 知道他们喜欢大数，所以为他们设计了一个计数游戏。 首先 Tadd 会说数 1，然后 Todd 必须说接下来的两个数（2 和 3）， 然后 Tucker 必须说接下来的三个数（4，5，6） ， 然后 Tadd 必须说接下来的四个数（7，8，9，10），这个过程继续在三个孩子之间按此顺序轮换，每个孩子都比前面的孩子多说一个数，直到说出 10, 000 为止。 问 Tadd 说出的第 2019 个数是多少？

(A) 5743 **(B)** 5885 **(C)** 5979 **(D)** 6001 **(E)** 6011

24. Let p, q, and r be the distinct roots of the polynomial $x^3-22x^2+80x-67$. It is given that there exist real numbers A, B, and C such

$$\frac{1}{s^3-22s^2+80s-67}=\frac{A}{s-p}+\frac{B}{s-q}+\frac{C}{s-r}$$

that for all $s\notin\{p,q,r\}$. What is $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}$?

设 p，q，和 r 是多项式$x^3-22x^2+80x-67$的三个不同的实根。 存在正实数 A， B，和 C 使得

$$\frac{1}{s^3-22s^2+80s-67}=\frac{A}{s-p}+\frac{B}{s-q}+\frac{C}{s-r}$$

对所有$s\notin\{p,q,r\}$的实数 s 成立。 问 $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}$是多少

(A) 243 **(B)** 244 **(C)** 245 **(D)** 246 **(E)** 247

25. For how many integers n between 1 and 50, inclusive, is

$$\frac{(n^2-1)!}{(n!)^n}$$

an integer? (Recall that 0! = 1.)

在从 1 到 50 的整数（包括首尾两数）n 中，有多少个数使得

n

$$\frac{(n^2-1)!}{(n!)^n}$$

是整数？（注意 $0!=1$。）

(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

参考答案

1. C
2. A
3. D
4. B
5. D
6. C
7. C
8. C
9. B
10. C
11. C
12. E
13. D
14. D
15. E
16. A
17. D
18. D
19. B
20. B
21. D
22. B
23. C
24. B
25. D