2018 AIME II

1. Points A, B, and C lie in that order along a straight path where the distance from A to C is 1800 meters. Ina runs twice as fast as Eve, and Paul runs twice as fast as Ina. The three runners start running at the same time with Ina starting at A and running toward C, Paul starting at B and running toward C, and Eve starting at C and running toward A. When Paul meets Eve, he turns around and runs toward A. Paul and Ina both arrive at B at the same time. Find the number of meters from A to B.

   A，B和C點按此順序在一條直線路徑上，從A到C的距離是1800米。Ina跑步的速度是Eve的兩倍，Paul跑步的速度是Ina的兩倍。這三名跑者同時出發，Ina從A跑向C，Paul從B跑向C，而Eve從C跑向A。當Paul遇到Eve時，他則轉身跑向A。 Paul和Ina同時到達B。求從A到B的距離的米數。

2. Let $a_0 = 2$, $a_1 = 5$, and $a_2 = 8$, and for $n > 2$ define a_n recursively to be the remainder when $4(a_{n-1} + a_{n-2} + a_{n-3})$ is divided by 11. Find $a_{2018} \cdot a_{2020} \cdot a_{2022}$.

   設$a_0 = 2$，$a_1 = 5$，和$a_2 = 8$，並且對於$n > 2$，遞推的定義a_n 為$4(a_{n-1} + a_{n-2} + a_{n-3})$除以11的餘數。求 $a_{2018} \cdot a_{2020} \cdot a_{2022}$。

3. Find the sum of all positive integers $b < 1000$ such that the base-b integer 36_b is a perfect square and the base-b integer 27_b is a perfect cube.

   正整數$b < 1000$滿足在b進位製表達下，整數36_b是一個完全平方數，整數27_b是一個完全立方數。求所有這樣正整數b的和。

4. In equiangular octagon $CAROLINE$, $CA = RO = LI = NE = \sqrt{2}$ and $AR = OL = IN = EC = 1$. The self-intersecting octagon $CORNELIA$ encloses six non-overlapping triangular regions. Let K be the area enclosed by $CORNELIA$, that is, the total area of the six triangular regions. Then $K = \frac{a}{b}$, where a and b are relatively prime positive integers. Find $a + b$.

   在等角的八邊形$CAROLINE$中，$CA = RO = LI = NE = \sqrt{2}$，和$AR = OL = IN = EC = 1$。自相交的八邊形$CORNELIA$ 圍出了六個不重疊的三角形區域。令K是$CORNELIA$所圍出的面積，即六個三角形區域的面積總和。設$K = \frac{a}{b}$，其中a和b是兩個互質的正整數。求$a + b$。

5. Suppose that x, y, and z are complex numbers such that $xy = -80 - 320i$, $yz = 60$, and $zx = -96 + 24i$, where $i = \sqrt{-1}$. Then there are real numbers a and b such that $x + y + z = a + bi$. Find $a^2 + b^2$.

   假設x，y和z是複數，滿足$xy = -80 - 320i$，$yz = 60$，和 $zx = -96 + 24i$，其中$i = \sqrt{-1}$。於是有實數a和b滿足$x + y + z = a + bi$。求$a^2 + b^2$。

6. A real number a is chosen randomly and uniformly from the interval $[-20, 18]$. The probability that the roots of the polynomial

$$x^4 + 2ax^3 + (2a - 2)x^2 + (-4a + 3)x - 2$$

are all real can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

實數a隨機均勻的取自區間$[-20, 18]$。多項式

$$x^4 + 2ax^3 + (2a - 2)x^2 + (-4a + 3)x - 2$$

的所有根都為實數的概率可以被寫作$\frac{m}{n}$，其中m和n是兩個互質的正整數。求$m + n$。

7. Triangle ABC has side lengths $AB = 9$, $BC = 5\sqrt{3}$, and $AC = 12$. Points $A = P_0, P_1, P_2, \ldots, P_{2450} = B$ are on segment $\overline{AB}$ with P_k between P_{k-1} and P_{k+1} for $k = 1, 2, \ldots, 2449$, and points $A = Q_0, Q_1, Q_2, \ldots, Q_{2450} = C$ are on segment $\overline{AC}$ with Q_k between Q_{k-1} and Q_{k+1} for $k = 1, 2, \ldots, 2449$. Furthermore, each segment $\overline{P_kQ_k}$, $k = 1, 2, \ldots, 2449$, is parallel to $\overline{BC}$. The segments cut the triangle into 2450 regions, consisting of 2449 trapezoids and 1 triangle. Each of the 2450 regions has the same area. Find the number of segments $\overline{P_kQ_k}$, $k = 1, 2, \ldots, 2450$, that have rational length.

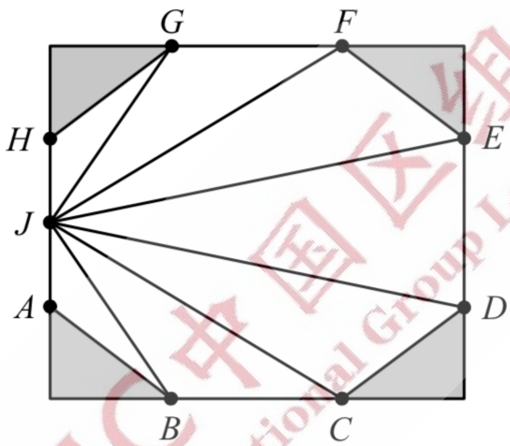
三角形ABC的邊長為$AB = 9$，$BC = 5\sqrt{3}$，和$AC = 12$。點$A = P_0, P_1, P_2, \ldots, P_{2450} = B$在線段$\overline{AB}$上，並且對於$k = 1, 2, \ldots, 2449$，$P_k$介於$P_{k-1}$與$P_{k+1}$之間；以及點$A = Q_0, Q_1, Q_2, \ldots, Q_{2450} = C$位於線段$\overline{AC}$上，並且對於$k = 1, 2, \ldots, 2449$，$Q_k$介於$Q_{k-1}$和$Q_{k+1}$之間。進一步的，每條線段$\overline{P_kQ_k}$，$k = 1, 2, \ldots, 2449$，都平行於$\overline{BC}$。這些線段將這個三角形劃分成了2450個區域，其中包括1個三角形和2449個梯形。這2450個區域中每個區域的面積都相等。在線段$\overline{P_kQ_k}$，$k = 1, 2, \ldots, 2450$中，求長度為有理數的線段的條數。

8. A frog is positioned at the origin in the coordinate plane. From the point (x, y), the frog can jump to any of the points $(x + 1, y)$, $(x + 2, y)$, $(x, y + 1)$, or $(x, y + 2)$. Find the number of distinct sequences of jumps in which the frog begins at $(0, 0)$ and ends at $(4, 4)$.

一隻青蛙在坐標系的原點。從點(x, y)，青蛙可以跳到$(x + 1, y)$，$(x + 2, y)$，$(x, y + 1)$，或者$(x, y + 2)$中的任意一點。求青蛙從$(0, 0)$到$(4, 4)$的不同跳躍序列的個數。

9. Octagon $ABCDEFGH$ with side lengths $AB = CD = EF = GH = 10$ and $BC = DE = FG = HA = 11$ is formed by removing four $6-8-10$ triangles from the corners of a 23×27 rectangle with side $\overline{AH}$ on a short side of the rectangle, as shown. Let J be the midpoint of $\overline{HA}$, and partition the octagon into 7 triangles by drawing segments $\overline{JB}$, $\overline{JC}$, $\overline{JD}$, $\overline{JE}$, $\overline{JF}$, and $\overline{JG}$. Find the area of the convex polygon whose vertices are the centroids of these 7 triangles.

八邊形$ABCDEFGH$各邊長為$AB = CD = EF = GH = 10$，和$BC = DE = FG = HA = 11$，它是從如圖所示的23×27的長方形去掉四個$6-8-10$的三角形而得到，$\overline{AH}$在長方形的短邊上。設J是$\overline{HA}$的中點，線段$\overline{JB}$，$\overline{JC}$，$\overline{JD}$，$\overline{JE}$，$\overline{JF}$，和$\overline{JG}$將八邊形分成了7個三角形。求頂點是這7個三角形的重心的凸多邊形的面積。



10. Find the number of functions $f(x)$ from $\{1,2,3,4,5\}$ to $\{1,2,3,4,5\}$ that satisfy $f(f(x)) = f(f(f(x)))$ for all x in $\{1,2,3,4,5\}$.

求所有滿足下述條件的從$\{1,2,3,4,5\}$到$\{1,2,3,4,5\}$的函數$f(x)$的個數：對於$\{1,2,3,4,5\}$中的全部x, 均有$f(f(x)) = f(f(f(x)))$。

11. Find the number of permutations of 1, 2, 3, 4, 5, 6 such that for each k with $1 \leq k \leq 5$, at least one of the first k terms of the permutation is greater than k.

求所有滿足下列條件的1，2，3，4，5，6的置換的個數：對於滿足$1 \leq k \leq 5$的每個k，置換的首k項中至少有一個數大於k。

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12. Let $ABCD$ be a convex quadrilateral with $AB = CD = 10$, $BC = 14$, and $AD = 2\sqrt{65}$. Assume that the diagonals of $ABCD$ intersect at point P, and that the sum of the areas of $\triangle APB$ and $\triangle CPD$ equals the sum of the areas of $\triangle BPC$ and $\triangle APD$. Find the area of quadrilateral $ABCD$.

$ABCD$是一個凸四邊形，$AB = CD = 10$，$BC = 14$，並且 $AD = 2\sqrt{65}$。設$ABCD$的對角線相交於P點，並且$\triangle APB$和$\triangle CPD$的面積之和等於$\triangle BPC$和$\triangle APD$的面積之和。求四邊形$ABCD$的面積。

13. Misha rolls a standard, fair six-sided die until she rolls $1-2-3$ in that order on three consecutive rolls. The probability that she will roll the die an odd number of times is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Misha投擲一個標準的公允的六個面的骰子直到連續三次擲出的結果順次是$1-2-3$為止。設她需要投擲奇數次的概率是$\frac{m}{n}$，其中m和n是互質的正整數。求$m+n$。

14. The incircle ω of $\triangle ABC$ is tangent to $\overline{BC}$ at X. Let $Y \neq X$ be the other intersection of $\overline{AX}$ and ω. Points P and Q lie on $\overline{AB}$ and $\overline{AC}$, respectively, so that $\overline{PQ}$ is tangent to ω at Y. Assume that $AP = 3$, $PB = 4$, $AC = 8$, and $AQ = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

$\triangle ABC$的內接圓ω與$\overline{BC}$相切於X。令$Y \neq X$是$\overline{AX}$與ω的另一交點。點P和點Q分別位於$\overline{AB}$和$\overline{AC}$上，使得$\overline{PQ}$與ω相切於Y。假設$AP = 3$，$PB = 4$，$AC = 8$並且$AQ = \frac{m}{n}$其中m與n是互質的正整數。求$m+n$。

15. Find the number of functions f from $\{0,1,2,3,4,5,6\}$ to the integers such that $f(0) = 0$, $f(6) = 12$, and

$$|x-y| \le |f(x)-f(y)| \le 3|x-y|$$

for all x and y in $\{0,1,2,3,4,5,6\}$.

函數f是從$\{0,1,2,3,4,5,6\}$到整數的映射，滿足$f(0) = 0$，$f(6) = 12$，並且對於$\{0,1,2,3,4,5,6\}$中的任何x和y，有

$$|x-y| \le |f(x)-f(y)| \le 3|x-y|。$$

求所有這樣的函數f的個數。